# Thomas Algorithm

# Objectives

- Solve a system of linear algebraic equations in tridiagonal form using Thomas Algorithm

- Let the system of linear algebraic equations be:
- A * X = B ; A – Matrix; X, B – Vectors
- $$\begin{pmatrix} -2.25 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -2.25 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2.25 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2.25 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2.25 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2.25 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -2.25 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \\ x4 \\ x5 \\ x6 \\ x7 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -100 \end{pmatrix}$$
- This is a tri-diagonal matrix with non zero numbers in the lower, middle and upper diagonal locations

- The above set of equations can be efficiently solved using Thomas Algorithm
- The A matrix is re-arranged as

- $\text{Anew} = \begin{pmatrix} 0 & -2.25 & 1 \\ 1 & -2.25 & 1 \\ 1 & -2.25 & 1 \\ 1 & -2.25 & 1 \\ 1 & -2.25 & 1 \\ 1 & -2.25 & 1 \\ 1 & -2.25 & 0 \end{pmatrix}$

- i.e., lower diagonal, middle, upper diagonal elements are arranged in the above form

- Algorithm :
- For the rows from 2 to n ( n- no. of rows of the matrix A)
- $d(i) = d(i) - \left(\frac{l(i)}{d(i-1)}\right) * u(i-1);$
- $b(i) = b(i) - \left(\frac{l(i)}{d(i-1)}\right) * b(i-1);$
- l – lower diagonal elements ;
- d – main diagonal elements
- u – upper diagonal elements
- b – right hand side vector

- Algorithm (Continued) :

- Solution:

- $x(n) = \left(\frac{b(n)}{d(n)}\right)$ ;

- for i varying from n-1 to 1

- $x(i) = \left(\frac{b(i) - u(i) * x(i+1)}{d(i)}\right)$

- Solution

- X = $\begin{pmatrix} -1.9668 \\ -4.4252 \\ -7.9899 \\ -13.5521 \\ -22.5024 \\ -37.0783 \\ -60.9237 \end{pmatrix}$

# Summary

In this video,

- We presented a procedure called Thomas Algorithm to solve a system of linear algebraic equations in tridiagonal form.
- The advantage of Thomas Algorithm is that the method is very efficient.
- We don't computer resources to store all the zero values existing in the non-tridiagonal locations.
- In the next video, we will show how to solve linear algebraic equations using iterative methods.